Assessment Schedule – 2007

Calculus: Integrate functions and use integrals to solve problems (90636)

Evidence Statement

	Achievement Criteria	Q.	Evidence	Code	Judgement	Sufficiency
ACHIEVEMENT	Integrate functions and use integrals to solve problems.	1a	$\frac{\text{cosec } 3x}{3}+\text{c}$ or $\frac{1}{3\sin 3x}+c$	A1	Or equivalent	**Achievement:** THREE of Code A **including** at least ONE Code A1 and ONE Code A2.
		1b	$\frac{5}{3}\ln\lvert \text{k}x\rvert$ or $\frac{5}{3}\ln\lvert x\rvert+c$ or $\ln\left\lvert \text{k}\sqrt[3]{x^5}\right\rvert$ etc	A1	Or equivalent. Absolute value sign not necessary.	
		2	$Vol=\pi\int_0^4\left(\frac{x^2}{12}+1\right)^2 dx$ $=\pi\int_0^4\left(\frac{x^4}{144}+\frac{x^2}{6}+1\right)dx$ $=\pi\left[\frac{x^5}{720}+\frac{x^3}{18}+x\right]_0^4$ $=\pi(1.42+3.55+4)$ $=\frac{404\pi}{45}=8\frac{44}{45}\pi=8.977\pi=28.20$	A1 or A2	Must show integrated function. Must show ∂ at least once Or equivalent.	
		3	$\int 4y\text{d}y=\int\cos 2x\text{d}x$ $2y^2=\frac{\sin 2x}{2}+\text{c}$ when $x=\frac{\partial}{12}, y=2$ then c = 7.75 $2y^2=\frac{\sin 2x}{2}+7.75$ or $8y^2 = 2\sin 2x + 31$	A1 or A2	Or equivalent.	
		4	$v=\int\frac{320\,000}{16\,000-400t}\text{d}t$ $=-800\ln\lvert 16\,000-400t\rvert+\text{c}$ when $t=0$, $v=0$ then c = 7744.3 or c = 800 ln 16000 $v=-800\ln\lvert 16\,000-400t\rvert+7744.3$ v(30)= -800 ln 4000 + 7744.3 = 1109.035 or $v=\int\frac{800}{40-t}dt = -800\ln\lvert 40-t\rvert+c$ when $t=0$, $v=0$ then c = 2951.1 v(30) = -800 ln 10 + 2951.1 = 1109.035 For 1600 – 40t, c = 5902.2 For 160 – 4t, c = 4060.1	A1 or A2	Must show equation for v. Or equivalent.	

	Achievement Criteria	No.	Evidence	Code	Judgement	Sufficiency
MERIT	Use advanced integration techniques to find integrals and solve problems.	5	$\frac{1}{3}\ln\left\|e^{3x}+9x\right\|+c$ or $\frac{1}{3}\ln\left\|k\left(e^{3x}+9x\right)\right\|$	A1 M	Absolute value signs not needed.	**Merit:** Achievement **Plus** THREE of Code M **OR** FOUR of Code M
		6	$V=\int\frac{6400}{\sqrt{1+20t}}\mathrm{d}t$ $=640\sqrt{1+20t}+c$ when $t=0$, $V=840$ then $c=200$ $V=640\sqrt{1+20t}+200$ When t = 6, $V=640\sqrt{1+120}+200$ $=7240\text{ cm}^3$	A1 or A2 M	Must show integrated function. Units not required.	
		7	Area $=\int_0^{\frac{\pi}{6}}(2\sin 5x\cos 3x)\mathrm{d}x$ $=\int_0^{\frac{\pi}{6}}(\sin 8x+\sin 2x)\mathrm{d}x$ $=\left[-\frac{\cos 8x}{8}-\frac{\cos 2x}{2}\right]_0^{\frac{\pi}{6}}$ $=\frac{1}{16}-\frac{1}{4}+\frac{1}{8}+\frac{1}{2}$ $=-\frac{3}{16}+\frac{5}{8}=-0.1875+0.625$ $=\frac{7}{16}=0.4375$	A1 or A2 M	Must show integrated function. Or equivalent.	
		8	$\frac{\mathrm{d}P}{\mathrm{d}t}=kP$ $\int\frac{1}{P}\mathrm{d}P=\int k\mathrm{d}t$ $\ln\left\|cP\right\|=kt$ $P=Ae^{kt}$ (0,100) gives $A=100$ (1,105) gives $k=\ln 1.05=0.04879$ $P=100e^{\ln 1.05t}$ When $P=200$, $t=\frac{\ln 2}{\ln 1.05}$ Population will double in 14.2 years. (Double to 210 gives 15.2 years)	A2 M	Or equivalent.	

	Achievement Criteria	No.	Evidence	Code	Judgement	Sufficiency
EXCELLENCE	Solve more complex integration problem(s).	9	$y = 9 - x^2$ $x^2 = 9 - y$ $Vol = \pi\int_0^6 (9-y)\,dy$ $= \pi\left[9y - \frac{y^2}{2}\right]_0^6$ $= 36\pi = 113.1$ cubic units Inverted shape $y = x^2$ Same volume of liquid. $36\pi = \pi\int_0^h y\,dy$ $= \partial\left[\frac{y^2}{2}\right]_0^h$ $36 = \frac{h^2}{2}$ $h = \sqrt{72} = 8.485$ cm or $6\sqrt{2}$ cm	A2 M or E	Units not required. Must use ∂ correctly Must show both integrals Or equivalent.	**Excellence:** Merit **plus** Code E

Question 5 by substitution

$$u = e^{3x} + 9x$$
$$\frac{du}{dx} = 3e^{3x} + 9$$
$$= 3\left(e^{3x} + 3\right)$$
$$\left(e^{3x} + 3\right)dx = \frac{du}{3}$$
$$\int \frac{e^{3x} + 3}{e^{3x} + 9x}\,dx = \int \frac{1}{u}\frac{du}{3}$$
$$= \frac{1}{3}\ln u$$
$$= \frac{1}{3}\ln\left|k(e^{3x} + 9)\right|$$

Question 6 by substitution

$$V = \int\left(\frac{6400}{\sqrt{1+20t}}\right)dt$$

$$u = 1 + 20t$$
$$du = 20dt$$
$$dt = \frac{du}{20}$$

$$\mathrm{V} = \int \frac{6400}{\mathrm{u}^{\frac{1}{2}}}\frac{\mathrm{du}}{20}$$
$$= \int 320\mathrm{u}^{-\frac{1}{2}}\mathrm{du}$$
$$= 620\mathrm{u}^{\frac{1}{2}} + \mathrm{c}$$
$$= 640\sqrt{1+20\mathrm{t}} + \mathrm{c}$$

$$V = \int\left(\frac{6400}{\sqrt{1+20t}}\right)dt$$

$$\mathrm{u} = \sqrt{(1+20\mathrm{t})}$$
$$\mathrm{u}^2 = 1 + 20\mathrm{t}$$
$$2\mathrm{udu} = 20\mathrm{dt}$$
$$\mathrm{dt} = \frac{\mathrm{udu}}{10}$$

$$\mathrm{V} = \int \frac{6400}{\mathrm{u}}\frac{\mathrm{udu}}{10}$$
$$= \int 640\mathrm{du}$$
$$= 640\mathrm{u} + \mathrm{c}$$
$$= 640\sqrt{1+20\mathrm{t}} + \mathrm{c}$$

Judgement Statement

Achievement	Achievement with Merit	Achievement with Excellence
Integrate functions and use integrals to solve problems. 3 × A including at least 1 × A1 *and* 1 × A2	Use advanced integration techniques to find integrals and solve problems. **Achievement** *plus* 3 × M *or* 4 × M	Solve more complex integration problems(s). **Merit** *plus* 1 × E

The following Mathematics specific marking conventions may also have been used when marking this paper:

- Errors are circled.
- Omissions are indicated by a caret (∧).
- **NS** may have been used when there was not sufficient evidence to award a grade.
- **CON** may have been used to indicate 'consistency' where an answer is obtained using a prior, but incorrect answer and **NC** if the answer is not consistent with wrong working.
- **CAO** is used when the 'correct answer only' is given and the assessment schedule indicates that more evidence was required.
- # may be used when a correct answer is obtained but then further (unnecessary) working results in an incorrect final answer being offered.
- **RAWW** indicates right answer, wrong working.
- **R** for 'rounding error' and **PR** for 'premature rounding' resulting in a significant round-off error in the answer (if the question required evidence for rounding).
- **U** for incorrect or omitted units (if the question required evidence for units).
- **MEI** may have been used to indicate where a minor error has been made and ignored.